Calculus

LIMITS & DERIVATIVES CHEAT SHEET

PROPERTIES OF LIMITS

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \quad if \ \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$$

LIMIT EVALUATIONS AT ±∞

$$\lim_{x\to+\infty} e^x = \infty \text{ and } \lim_{x\to-\infty} e^x = 0$$

$$\lim_{x\to\infty} \ln x = \infty \text{ and } \lim_{x\to 0^+} \ln x = -\infty$$

$$\text{if r} > 0\text{:} \ \lim_{x\to\infty}\frac{c}{x^r} = 0$$

$$\text{if r} > 0 \ \& \ \{\forall x > 0 | x^r \in \mathbb{R}\}: \ \lim_{x\to-\infty}\frac{c}{x^r} = 0$$

$$\lim_{x\to\pm\infty} x^r = \infty \text{ for even r}$$

$$\lim_{x\to+\infty} x^r = \infty \text{ and } \lim_{x\to-\infty} x^r = -\infty \text{ for odd r}$$

L'HOPITAL'S RULE

$$\text{If } \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty} \text{ then } \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

DERIVATIVE DEFINITION

$$\frac{d}{dx}[f(x)] = f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

PRODUCT RULE

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

QUOTIENT RULE

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

CHAIN RULE

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

BASIC PROPERTIES OF DERIVATIVES

$$[cf(x)]' = c[f'(x)]$$

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

COMMON DERIVATIVES

$\frac{d}{dx}(x) = 1$	$\frac{d}{dx}[af(x)] = a\frac{d}{dx}[f(x)]$
$\frac{d}{dx}(ax) = a$	$\frac{d}{dx}(ax^n) = nax^{n-1}$
$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$

$$\frac{d}{dx}\left[\frac{1}{x^n}\right] = -nx^{-(n+1)} = -\frac{n}{x^{n+1}}$$

DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

$\frac{d}{dx}[\sin(x)] = \cos x$	$\frac{d}{dx}[\sec(x)] = \sec x \tan x$
$\frac{d}{dx}[\cos(x)] = -\sin x$	$\frac{d}{dx}[\csc(x)] = -\csc x \cot x$
$\frac{d}{dx}[\tan(x)] = \sec^2 x$	$\frac{d}{dx}[\cot(x)] = -\csc^2 x$

DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

$\frac{d}{dx}[e^x] = e^x$	$\frac{d}{dx}[a^x] = a^x \ln a$
$\frac{d}{dx}[\ln\|x\|] = \frac{1}{x}$	$\frac{d}{dx}[\ln x] = \frac{1}{x}, x > 0$
$\frac{d}{dx}[\log_a x] = \frac{1}{x\ln a}$	$\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}$
$\frac{d}{dx}[e^{f(x)}] = f'(x)e^{f(x)}$	$\frac{d}{dx}[a^{f(x)}] = a^{f(x)}\ln a\, f'(x)$

$$\frac{d}{dx}[f(x)^{g(x)}] = f(x)^{g(x)}\left(\frac{g(x)f'(x)}{f(x)} + \ln(f(x))\,g'(x)\right)$$

DERIVATIVES OF INVERSE TRIG FUNCTIONS

$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}[\sec^{-1} x] = \frac{1}{\|x\|\sqrt{x^2-1}}$
$\frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}[\csc^{-1} x] = -\frac{1}{\|x\|\sqrt{x^2-1}}$
$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$	$\frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1+x^2}$

DERIVATIVES OF HYPERBOLIC FUNCTIONS

$\frac{d}{dx}[\sinh x] = \cosh x$	$\frac{d}{dx}[\text{sech}\, x] = -\coth x \,\text{csch}\, x$
$\frac{d}{dx}[\cosh x] = \sinh x$	$\frac{d}{dx}[\text{csch}\, x] = -\tanh x \,\text{sech}\, x$

$$\frac{d}{dx}[\tanh x] = 1 - \tanh^2 x$$

$$\frac{d}{dx}[\coth x] = -1 - \coth^2 x$$

Calculus
INTEGRATION CHEAT SHEET

INTEGRATION PROPERTIES

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

where f is continuous on $[a,b]$ and $f' = F$

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x$$

where $\Delta x = \dfrac{b-a}{n}$ *and* $x_k = a + k\Delta x$

COMMON INTEGRALS

$\int k\,dx = kx + C$	$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$
$\int \dfrac{1}{x}\,dx = \ln\lvert x\rvert + C$	$\int \ln x\,dx = x\ln x - x + C$
$\int e^x\,dx = e^x + C$	$\int b^x\,dx = \dfrac{b^x}{\ln b} + C$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

INTEGRATION BY SUBSTITUTION

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

where $u = g(x)$ and $du = g'(x)\,dx$

INTEGRALS OF TRIGONOMETRIC FUNCTIONS

$\int \cos x\,dx = \sin x + C$	$\int \csc x\cot x\,dx = -\csc x + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sec x\tan x\,dx = \sec x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \csc x\,dx = \ln\lvert\csc x - \cot x\rvert + C$
$\int \sinh x\,dx = \cosh x + C$	$\int \cosh x\,dx = \sinh x + C$

$$\int \tan x\,dx = \ln\lvert\sec x\rvert + C$$

$$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

PARTIAL FRACTIONS

$$\frac{R(x)}{Q(x)} = \frac{A_1}{(a_1x+b_1)} + \frac{A_2}{(a_2x+b_2)} + \cdots + \frac{A_n}{(a_nx+b_n)},$$

where $Q(x) = (a_1x+b_1)(a_2x+b_2)\ldots(a_nx+b_n)$

$$\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{(a_1x+b_1)^2} + \cdots + \frac{A_n}{(a_1x+b_1)^n}$$

where a linear factor of $Q(x)$ is repeated n times

$$\frac{R(x)}{Q(x)} = \frac{Ax+B}{ax^2+bx+c}$$

where $Q(x)$ has a factor ax^2+bx+c, where $b^2-4ac<0$

$$\frac{R(x)}{Q(x)} = \frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

where $Q(x)$ has a factor (ax^2+bx+c), where $b^2-4ac<0$

INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du, \qquad \text{where } v = \int dv$$

or $\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$

INTEGRALS OF SYMMETRIC FUNCTIONS

If f is even $[f(-x) = f(x)]$, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

If f is odd $[f(-x) = -f(x)]$, then $\int_{-a}^{a} f(x)\,dx = 0$

Calculus

INTEGRATION & MULTIVARIATE CALCULUS CHEAT SHEET

APPLICATIONS OF INTEGRATION

AREA BETWEEN 2 CURVES	$A = \int_a^b \lvert f(x) - g(x) \rvert \, dx$
VOLUME DEFINITION	$\lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\Delta x = \int_a^b A(x)\, dx$

INTEGRATION BY TRIGONOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EVALUATION	IDENTITY USED
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\, d\theta$	$\sqrt{a^2 - a^2\sin^2\theta} = a\cos\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\, d\theta$	$\sqrt{a^2\sec^2\theta - a^2} = a\tan\theta$	$\sec^2\theta - 1 = \tan^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\, d\theta$	$\sqrt{a^2 + a^2\tan^2\theta} = a\sec\theta$	$1 + \tan^2\theta = \sec^2\theta$

STRATEGY FOR EVALUATING $\int \sin^m x \cos^n x\, dx$

If n is odd

$$\int \sin^m x \cos^{2k+1} x\, dx = \int \sin^m x\, (\cos^2 x)^k \cos x\, dx$$

$$= \int \sin^m x\, (1 - \sin^2 x)^k \cos x\, dx$$

Then substitute: $u = \sin x$

If m is odd

$$\int \sin^{2k+1} x \cos^n x\, dx = \int (\sin^2 x)^k \cos^n x \sin x\, dx$$

$$= \int (1 - \cos^2 x)^k \cos^n x \sin x\, dx$$

Then substitute: $u = \cos x$

If n and m are even

Use the half angle identities:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \text{ and } \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

or this identity: $\sin x \cos x = \frac{1}{2}\sin 2x$

STRATEGY FOR EVALUATING $\int \tan^m x \sec^n x\, dx$

If n is even

$$\int \tan^m x \sec^{2k} x\, dx = \int \tan^m x\, (\sec^2 x)^{k-1} \sec^2 x\, dx$$

$$= \int \tan^m x\, (1 + \tan^2 x)^{k-1} \sec^2 x\, dx$$

Then substitute: $u = \tan x$

If m is odd

$$\int \tan^{2k+1} x \sec^n x\, dx$$

$$= \int (\tan^2 x)^k \sec^{n-1} x \sec x \tan x\, dx$$

$$= \int (\sec^2 x - 1) \sec^{n-1} x \sec x \tan x\, dx$$

Then substitute: $u = \sec x$

PRODUCT IDENTITIES

$\int \sin mx \cos nx\, dx$	$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
$\int \sin mx \sin nx\, dx$	$\sin A \sin B = \frac{1}{2}[\cos(a - B) - \cos(A + B)]$
$\int \cos mx \cos nx\, dx$	$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

DERIVATIVES OF VECTOR FUNCTIONS

$$\frac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u}'(t) + \mathbf{v}'(t)$$

$$\frac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u}'(t)$$

$$\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$$

$$\frac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$$

$$\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$$

$$\frac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u}'(f(t)), \qquad \text{(chain rule)}$$

DEFINITE INTEGRAL OF A VECTOR FUNCTION

$$\int_a^b \mathbf{r}(t)\, dt$$

$$= \left(\int_a^b f(t)\, dt\right)\mathbf{i} + \left(\int_a^b g(t)\, dt\right)\mathbf{j} + \left(\int_a^b h(t)\, dt\right)\mathbf{k}$$