

Trigonometry

LAWS & IDENTITIES CHEAT SHEET

ODD & EVEN IDENTITIES

$\sin(-\theta) = -\sin(\theta)$	$\csc(-\theta) = -\csc(\theta)$
$\cos(-\theta) = \cos(\theta)$	$\sec(-\theta) = \sec(\theta)$
$\tan(-\theta) = -\tan(\theta)$	$\cot(-\theta) = -\cot(\theta)$

DOUBLE ANGLE IDENTITIES

HALF ANGLE IDENTITIES

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

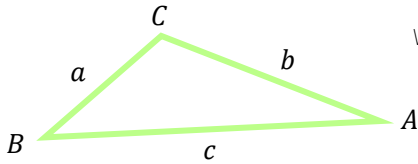
$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ \cos(2\theta) &= 2 \cos^2 \theta - 1 \\ \cos(2\theta) &= 1 - 2 \sin^2 \theta \end{aligned}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$



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LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc \cos \hat{A} \quad \hat{A} = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$b^2 = a^2 + c^2 - 2ac \cos \hat{B} \quad \hat{B} = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ab}\right)$$

$$c^2 = a^2 + b^2 - 2ab \cos \hat{C} \quad \hat{C} = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ac}\right)$$

LAW OF SINES

$$\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b} = \frac{\sin \hat{C}}{c}$$

AREA OF A TRIANGLE

$$\text{Area} = \frac{1}{2} ab \sin \hat{C}$$

$$\text{Area} = \frac{1}{2} bc \sin \hat{A}$$

$$\text{Area} = \frac{1}{2} ac \sin \hat{B}$$

PYTHAGOREAN IDENTITIES

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

SQUARE IDENTITIES

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

PRODUCT / SUM IDENTITIES

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

SUM IDENTITIES

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

DIFFERENCE IDENTITIES

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

CO-FUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

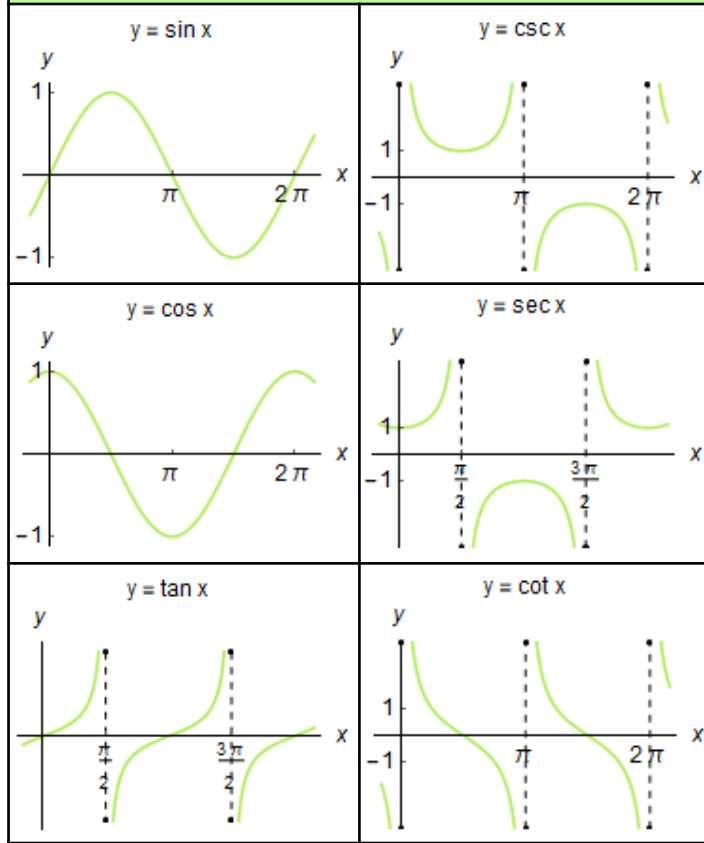
SPECIAL ANGLES

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	-

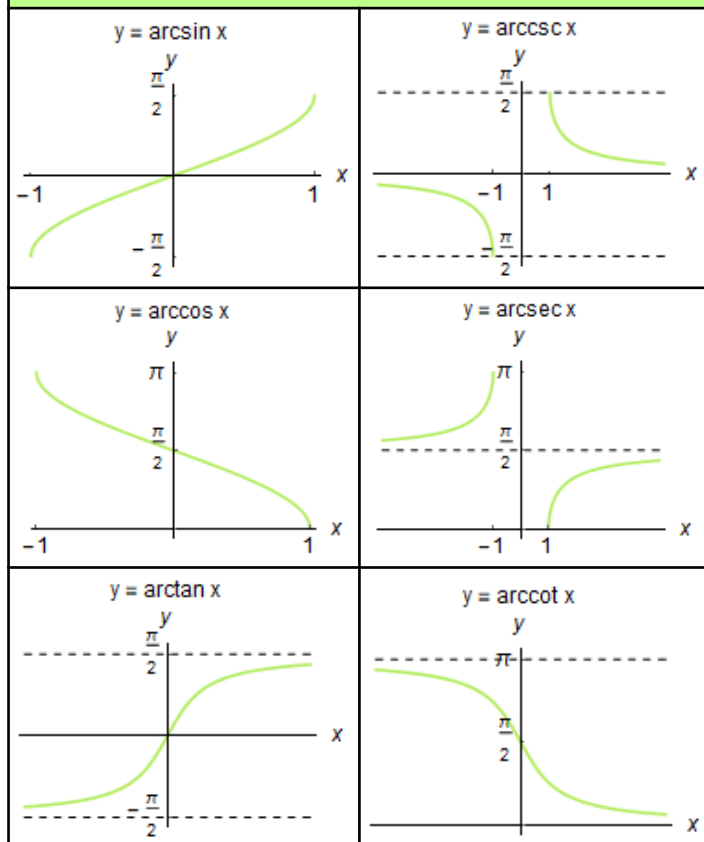
Trigonometry

TRIGONOMETRIC FUNCTIONS CHEAT SHEET

GRAPHS OF TRIGONOMETRIC FUNCTIONS



GRAPHS OF INVERSE TRIGONOMETRIC FUNCTIONS



INVERSE TRIGONOMETRIC FUNCTIONS

$$\arcsin x = \sin^{-1} x = y \Leftrightarrow \sin y = x \text{ and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$\arccos x = \cos^{-1} x = y \Leftrightarrow \cos y = x \text{ and } 0 \leq y \leq \pi$$

$$\arctan x = \tan^{-1} x = y \Leftrightarrow \tan y = x \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2}$$

DEGREES & RADIAN

$$\theta = \alpha \times \frac{180}{\pi}$$

$$\alpha = \theta \times \frac{\pi}{180}$$

HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\text{csch } x = \frac{1}{\sinh x}$$

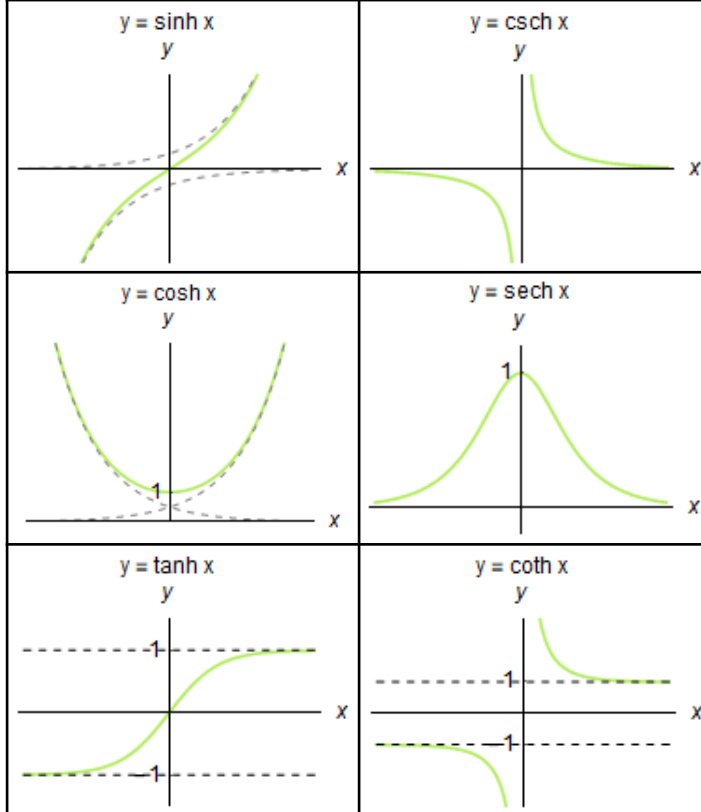
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\text{sech } x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\text{coth } x = \frac{\cosh x}{\sinh x}$$

GRAPHS OF HYPERBOLIC FUNCTIONS



INVERSE HYPERBOLIC FUNCTIONS

$$y = \sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$y = \cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), \quad y \geq 0$$

$$y = \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

$a \times \sin[b(x - c)] + d$

a : amplitude

c : horizontal shift

$b \div 360^\circ$: period

d : vertical shift